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Combining equilibrium, resampling, and analyst's views in portfolio optimization

José Luiz Barros Fernandes [a,b,*], José Renato Haas Ornelas [a], Oscar Augusto Martínez Cusicanqui [c]

[a] Banco Central do Brasil, Ed-Sede – 17th floor, Brasília 70074-900, DF, Brazil
[b] Universidade Católica de Brasília, Brasília 71966-700, DF, Brazil
[c] Banco Central de Bolivia, Ayacucho y Mercado, La Paz, Bolivia

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ABSTRACT

This paper proposes the use of a portfolio optimization methodology which combines features of equilibrium models and investor's views as in Black and Litterman (1992), and also deals with estimation risk as in Michaud (1998). In this way, our combined methodology is able to meet the needs of practitioners for stable and diversified portfolio allocations, while it is theoretically grounded on an equilibrium framework. We empirically test the methodology using a comprehensive sample of developed countries fixed income and equity indices, as well as sub-samples stratified by geographical region, time period, asset class and risk level. In general, our proposed combined methodology generates very competitive portfolios when compared to other methodologies, considering three evaluation dimensions: financial efficiency, diversification, and allocation stability. By generating financially efficient, stable, and diversified portfolio allocations, our methodology is suitable for long-term investors such as Central Banks and Sovereign Wealth Funds.

1. Introduction

Portfolio optimization methodologies play a central role in strategic asset allocation (SAA) where it is desirable to have portfolios that are efficient, diversified, and stable. Since the development of the traditional mean–variance approach of Markowitz (1952), many improvements have been made to overcome problems, such as lack of diversification and strong sensitivity of optimal portfolio weights to expected returns.

The Black and Litterman (1992) model (hereafter BL) is among the most used approaches. The main idea of this model is that expected returns are the result of two important sources of information: market information in the form of equilibrium returns (implicit returns that clear out the outstanding market allocation), and analysts' views which tilt the market portfolio to another diversified portfolio compatible with investor beliefs. In this fashion, portfolio managers get an intuitive but formal model to generate optimal allocation.

However, while the BL model offers a very useful and intuitive approach to deal with asset allocation, the inputs considered for the calculation of equilibrium returns are subject to estimation error. Michaud (1998) proposed the use of resampling to deal with estimation error, which is an important source of lack of diversification in mean–variance portfolio. This technique considers that data come from a stochastic process instead of being a deterministic input as in Markowitz (1952).

This paper proposes the use of a portfolio optimization methodology which combines features of both BL and resampling methodologies. This methodology allows a novel combination of equilibrium and investor's views as in BL, and at same time, deals with estimation risk as in Michaud (1998). Thus, it generates robust and diversified optimal allocations which are desirable properties for long-term investors such as Central Banks and Sovereign Wealth Funds. We empirically test this methodology using a sample of fixed income and equity indices, achieving very supportive results. We find strong evidence supporting the use of resampling techniques to improve standard models like BL and Markowitz. In general, our proposed combined methodologies, both with and without views, generated very competitive portfolios compared to the other methodologies, considering the three evaluation dimensions: financial efficiency, diversification, and allocation stability.

The remainder of this paper is as follows. Next section offers a brief literature review over asset allocation methodologies. The third section describes the Black-Litterman-Resampling combined methodology. The fourth section describes the empirical study,

* Corresponding author at: Banco Central do Brasil, Ed-Sede – 17th floor, Brasília 70074-900, DF, Brazil. Tel.: +55 61 3414 2329; fax: +55 61 3414 3245.

E-mail addresses: jluiz.fernandes@bcb.gov.br (J.L. Barros Fernandes), jrenato.ornelas@bcb.gov.br (J.R. Haas Ornelas), omartinez@bcb.gob.bo (O.A. Martínez Cusicanqui).

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including data, implementation and initial results. Section 6 presents the robustness checks and Section 7 concludes the paper.

2. Literature review

The seminal work of Markowitz (1952) provided the first model for asset allocation, arguing that once expected returns and their joint variance were defined, a set of efficient portfolios could be generated and investors would choose the allocation according to their needs. Basically the approach can be summarized as follows:

$$\text{Min}(1/2)\mathbf{a}^T\mathbf{Va} \tag{1}$$

subject to

$$E[\mathbf{R}_a] = \mathbf{a}^T\mathbf{X},$$

where $\mathbf{X}$ is the vector of expected excess asset returns, $\mathbf{a}$ is the vector of allocations, and $\mathbf{V}$ is the variance–covariance matrix of asset returns. Despite its mathematical simplicity, this model typically generates concentrated allocations which heavily depend on expected returns estimation. Resampling techniques (Michaud, 1998) were developed as a way to deal with estimation error. Markowitz recognized that resampling methods could be used to obtain better estimates for the inputs of the mean–variance optimization (Markowitz and Usmen, 2003).

Jorion (1991) used the Bayesian approach to overcome the weakness of expected returns estimated solely by sample information. He proposed an estimator obtained by "shrinking" the mean values toward a common value, chosen to be the expected return for the minimum variance portfolio. Kempf et al. (2002) applied Bayesian methods and considered estimation risk as a second source of risk, determined by the heterogeneity of the market, which is represented by the standard deviation of the expected returns across risky assets. Both methods proved to generate better out-of-sample estimates for expected returns (as opposed to in-sample estimates), and also produced more diversified portfolios.

Black and Litterman (1992) built a bridge between statistical methods and expert judgment by recognizing that capital asset pricing model (CAPM) offers an appropriate starting point for expected excess returns. Thus, combing CAPM with investors' views would produce intuitive and diversified allocations. For that, BL assume that equilibrium returns (CAPM returns that clear out the market) are well described by the following relationship:

$$\mathbf{X} \sim N(\mathbf{\Pi}, \boldsymbol{\tau\Sigma}), \tag{2}$$

where $\mathbf{X}$ is the observed returns vector which is just a realization of the multivariate normal process with mean $\mathbf{\Pi}$ (equilibrium returns), covariance matrix $\sum$ and an scale parameter $\boldsymbol{\tau}$ which measures the degree of confidence the investor has on equilibrium estimates (the closer the parameter is to zero, the higher is the confidence in equilibrium estimates).

In addition to this, BL postulate that returns have another important source of information, coming from investor's views:

$$\mathbf{X} \sim N(\mathbf{Q}, \mathbf{\Omega}), \tag{3}$$

where $\mathbf{Q}$ denotes the vector of expected return views (this could be absolute or relative) and $\mathbf{\Omega}$ is the uncertainty in those views. Since $\mathbf{\Omega}$ is not an easy-to-obtain parameter, we employ the Idzorek (2004) approach which measures the uncertainty through a degree of confidence and implicitly calculates $\mathbf{\Omega}$. With both sources of information, the combined process is also a multivariate normal as follows:

$$\mathbf{X} \sim N([(\boldsymbol{\tau}\Sigma)^{-1} + \mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}]^{-1}[(\boldsymbol{\tau}\Sigma)^{-1}\Pi + \mathbf{P}^T\mathbf{\Omega}\mathbf{Q}], [(\boldsymbol{\tau}\Sigma)^{-1} + \mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}]^{-1}), \tag{4}$$

where $\mathbf{P}$ denotes the portfolio view matrix whose dimension is a function of the number of views (rows) and the number of assets (columns). Needless to say is that, since market capitalization offers a well-diversified portfolio, the optimal allocation (in general) will have this property with tilts reflecting investors' views introduced in the model.

Finally, Michaud (1998) adapted the resampling statistic technique to mean–variance optimization recognizing that return history is just a realization of the stochastic process behind it. Also, if stationarity holds and in a large sample environment, the point estimates could statistically resemble the true distribution parameters. Suppose that we have a vector of expected excess return $\mathbf{X_0}$ and a variance–covariance matrix denoted by $\mathbf{\Sigma_0}$ (both estimated with a sample of returns of length $\boldsymbol{k}$), assuming that returns come from a multivariate normal distribution (with parameters $(\mathbf{X_0}, \mathbf{\Sigma_0})$), the procedure resamples n times joint returns of length k and estimates different parameters $(\{(\mathbf{X_1}, \mathbf{\Sigma_1}), (\mathbf{X_2}, \mathbf{\Sigma_2}), \ldots, (\mathbf{X_n}, \mathbf{\Sigma_n})\})$ which allow us to obtain n efficient frontiers. For a given portfolio, the resampled weights are given by the average of portfolio weights of the n samples:

$$\mathbf{a_R} = \frac{\mathbf{1}}{\mathbf{n}} \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{a_i}, \tag{5}$$

where $\mathbf{a_R}$ is the vector of the asset's weights in the resampled portfolio, and $\mathbf{a_i}$'s are the weights of each of the n realizations.

Several out-of-sample evaluations have shown results in favor of resampling methodology, using different sets of data (see, for instance, Markowitz and Usmen, 2003; Wolf, 2006; Fernandes and Ornelas, 2009). However, these evaluations cannot give definitive conclusions in favor of using resampling, given sampling limitations. Nevertheless, Fernandes and Ornelas (2009) and Kohli (2005) point out that resampled portfolios have two desirable characteristics for long-term investors. First, it usually generates portfolios that have greater diversification as more assets enter into the solution than in classical mean–variance efficient portfolios. Second, the model exhibits smoother transitions and less sudden shifts in allocations as return expectations change, meaning that the transaction costs of rebalancing the portfolio are typically lower. Nevertheless, the traditional resampling methodology, considered as an ad hoc methodology, has been criticized because of its lack of a theoretical basis.

3. Description of the Black–Litterman-resampling methodology

Since the source of estimation error comes at the first part of the BL model and spreads out until the final results, we combine the BL model with resampling techniques. The combined Black-Litterman-Resampling could be summarized as follows:

1. Estimate the BL expected return vector $\mathbf{X}$ and the covariance matrix $\mathbf{\Sigma}$ from historical inputs and maybe also combining with analysts' views.
2. Resample from results obtained in Step 1, by taking n draws of length L from a multivariate normal distribution with return vector $\mathbf{X}$ and covariance matrix $\mathbf{\Sigma}$.
3. For each draw n, calculate the new expected return and variance matrix. Because estimation error is present, these resampling estimates are different from the ones calculated in Step 1.
4. For each of the n sets of expected returns and covariance matrix calculated in Step 3, calculate the efficient frontier using traditional Markowitz optimization. The output of this step will be a set of n efficient frontiers.
5. For each risk level, calculate the average portfolio weights across the n efficient frontiers. These weights define the portfolios of the BL Resampling frontier. The risk $\times$ return profile of the BLR can be calculated using the expected return vector $\mathbf{X}$ and the covariance matrix $\mathbf{\Sigma}$ from Step 1.

Table 1
Descriptive statistics.

Name	Market	Instrument type	Mean	Standard deviation	Skewness	Kurtosis
TBill 3M	USA	Bonds	4.56	0.66	−0.13	−0.60
US Govt	USA	Bonds	7.14	4.91	−0.05	0.57
Canada Govt	CAN	Bonds	9.80	9.10	−0.45	1.87
Australia Govt	AUS	Bonds	11.34	12.59	−0.69	1.83
Japan Govt	JAP	Bonds	8.39	13.01	0.51	1.63
Germany Govt	GER	Bonds	9.21	11.72	0.06	0.47
UK Govt	UKG	Bonds	9.62	11.93	0.14	0.73
Germany Equity	GER	Equity	10.32	21.00	−0.53	1.32
Australia Equity	AUS	Equity	15.18	23.11	−1.31	7.01
Japan Equity	JAP	Equity	6.68	23.16	0.37	0.58
Canada Equity	CAN	Equity	12.75	19.08	−0.88	3.65
UK Equity	UKG	Equity	12.00	17.94	−0.40	1.66
SP500	USA	Equity	11.66	15.89	−0.81	2.25
SP600 SMALL	USA	Equity	10.27	18.90	−0.98	3.25
SP400 MID	USA	Equity	11.35	16.93	−0.97	3.22
US Corporate	USA	Corporate	7.81	5.51	−0.74	4.04

This table presents descriptive statistics about returns from asset classes considered in the analysis. Calculations are done with monthly data. The mean and standard deviation are presented in annualized percentage points.

Finally, the Black–Litterman-resampling methodology overcomes the highly criticized weakness of the traditional resampling approach as being only an ad hoc methodology. This combined methodology has the theoretical background of an equilibrium model as in Black and Litterman (1992). At the same time, the use of resampling enriches the Bayesian BL approach to optimization, by recognizing estimation error.

4. Empirical study

4.1. Data and implementation

Our tests are based on monthly data of 15 indices of bonds and stocks from six developed countries. For bonds, we use six developed countries' government bond indices, and one US Corporate bond index from BofA Merrill Lynch. For equities, we use the Thomson Datastream market indices from five countries, and three US Equity market indices, divided by market capitalization: S&P500 Composite, S&P 400 Midcap and S&P600 Small Cap.

The period of the sample is from January, 1986, to December, 2009, with a total of 288 months. The sample has data from market capitalization and total return index levels. All total return time series are calculated on a US-dollar basis and we use the 3-month US T-Bill rate when calculating excess returns. We are aware that this assumption favors US assets. US dollar is typically the *numeraire* considered by global investors, such as Central Banks, pension funds, and multinational institutions. Table 1 presents the descriptive statistics of each asset class considered.

The return time series used present the usual financial statistical characteristics, presenting a positive risk premium, negative skewness, and excess kurtosis. Fig. 1 presents the risk return information for the considered asset classes. Note that, except for Japan equities, there is an upward slope of the tendency line indicating the positive risk premium.

The list of methodologies that we address in this paper is shown in Table 2. The first one is the traditional Markowitz (Mark) approach (Markowitz, 1952), which uses quadratic optimization considering historical data to estimate risk and return. Resampling (Res) is the methodology proposed in Michaud (1998).

Two methodologies are based on the Black and Litterman (1992) article. The first one considers just equilibrium risk and return estimates (BL), while the second (BLV) applies the complete BL model, which combines equilibrium estimates with analysts' views. In this case, we chose as analysts' views just the historical averages, with a confidence of 50% (we use Idzorek's (2004) framework to set the confidence level). It is important to highlight that we are not supporting any specific view, but just pointing out that our portfolio optimization techniques may be used considering any view methodology.

Finally, we have two methods that combine BL methodology with resampling, which were described in Section 3. The first one, BLRes, combines the BL pure equilibrium model with the resampling methodology. The second, BLVR, combines the BL equilibrium model with the same analysts' views of the BLV model and also applies the resampling technique.

We analyze the performance of these six optimization strategies on Table 2. We do an out-of-sample analysis with a rolling estimation window of 60 months. So the model's parameters are estimated using monthly return observations of the past 60 months. Then we estimate the efficient frontier and hold it for the next (e) months (results for $e = 6$ months are on Table 6 and results for $e = 12$ are on Table 4). Then we move e months forward the estimation window, and re-estimate the parameters and adjust the portfolio weights. We repeat this procedure until the end of the sample period. We evaluate the methodology's performance with a no short-selling constraint as is usual in the industry. We consider three levels of investors' risk preference[1] (low, medium, and high), in order to infer if the results are sensitive to investors' level of risk aversion. We use an estimation period of 60 months, and evaluation periods of 6 and 12 months.

To find out which methodology outperforms the others, we take into account three evaluation dimensions: financial performance, allocation stability, and diversification – as ideally an investor would prefer stable, diversified, and financially efficient portfolios. To access the financial performance of the strategies, we compute the traditional Sharpe ratio, and another measure that goes beyond the mean–variance world: the Farinelli-Tibilleti ratio (see, for instance, Farinelli et all, 2008). In order to evaluate the stability of the allocation weights generated by the methodology we calculate the average turnover[2] of the portfolio in the re-estimation events. To measure the diversification, we use the mean Herfindahl index, given by the sum of the squared asset allocation weights.

[1] These three levels of risk are based on the standard deviation of returns in the efficient frontiers estimated. We take the universe of all possible standard deviations in the frontiers, and split into 11 levels equally spaced. Our medium level of risk is the 6th level, while the low is the 3rd level and the high risk is the 9th level.

[2] The turnover is given by the sum of the purchases and sales divided by the value of the portfolio.

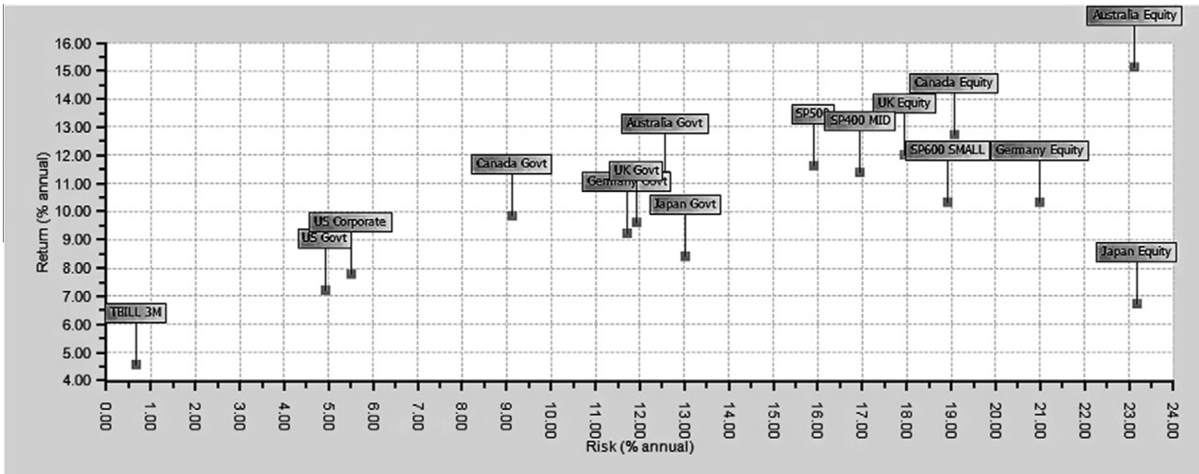

Fig. 1. Risk and return characteristics from asset classes included in the sample.

Table 2
List of methodologies.

Mnemonic	Methodology
Mark	Markowitz Portfolio Selection Model
Res	Portfolio Resampling Model
BL	Black & Litterman Pure Equilibrium Model
BLV	Black & Litterman Equilibrium Model with Analysts' Views
BLRes	Black & Litterman Pure Equilibrium Model with Resampling
BLVR	Black & Litterman Equilibrium Model with Analysts' Views and Resampling

4.2. Results

Table 3 presents the results of the Sharpe ratio, Farinelli-Tibiletti (hereafter FT) ratio, Herfindahl index, and turnover for each methodology considering three levels of risk for the optimal portfolio: low (panel A), medium (panel B), and high (panel C). For the FT ratio, we need to set two parameters, which define the risk tolerance of the investors. We considered three different parameterizations: for the low risk we used $p = 0.5$; $q = 2$; for the medium level of risk $p = 1$; $q = 1$ and for the high level of risk, $p = 2$; $q = 0.5$. Thus, for the low risk, we use a parameters consistent with a very risk-averse investor, and for the high risk, we use parameters for a less risk-averse investors.

The estimation period considered was 60 months and the evaluation period (e) was 6 months. So, the optimizer considers the previous 5 years of data to estimate the frontier portfolios and holds the allocation for the next 6 months, for which the returns are calculated. After that, the frontier is re-estimated again considering the previous 5 years of data and so on until the end of the sample period. Therefore, results of Table 3 are based on 47 portfolio optimization experiments for each methodology, since the holding period is 6 months.

Comparing first the Markowitz (Mark) results against resampling (Res), we can see that for the three levels of risk, the Res portfolio outperformed the Mark one in all three evaluation dimensions considered: more financially efficient, diversified, and stable. This is a well-known result in the literature when we compare the traditional Markowitz approach with resampling (Markowitz and Usmen, 2003). As resampling techniques typically generate more diversified portfolios, they tend to reduce turnover and increase financial out-of-sample efficiency.

When we compare the BL equilibrium with the BL equilibrium with resampling (BLRes) results, we find out more supportive numbers related to the use of resampling. For every level of risk,

Table 3
Optimization results with 6-months evaluation period.

	Mark	Res	BL	BLRes	BLV	BLVR
Panel A. Low risk level						
Sharpe ratio	0.539	0.573	0.562	**0.606**	0.526	0.589
Farinelli-Tibiletti	0.336	0.342	0.339	**0.355**	0.338	0.353
Herfindahl index	0.446	0.402	0.377	0.353	0.337	**0.336**
Turnover	0.308	0.257	0.242	0.213	0.281	**0.209**
Panel B. Medium risk level						
Sharpe ratio	0.367	0.428	0.402	**0.509**	0.388	0.503
Farinelli-Tibiletti	1.339	1.403	1.37	**1.492**	1.361	1.491
Herfindahl index	0.296	0.201	0.207	**0.103**	0.209	0.12
Turnover	0.577	0.38	**0.191**	0.196	0.494	0.256
Panel C. High risk level						
Sharpe ratio	0.346	0.383	0.174	0.3	0.318	**0.409**
Farinelli-Tibiletti	6.718	7.139	5.357	5.97	6.462	**7.358**
Herfindahl index	0.344	0.18	0.339	**0.113**	0.33	0.127
Turnover	0.715	0.469	**0.221**	0.237	0.596	0.313

Notes: This table presents the results of the Sharpe ratio, Farinelli-Tibiletti ratio, Herfindahl index, and turnover for each methodology considering three levels of risk for the optimal portfolio: low (panel A), medium (panel B), and high (panel C). The estimation period considered was 60 months and the evaluation period was 6 months. Best results for each evaluation criterion are shown in bold.

Table 4
Optimization results with 12-months evaluation period.

	Mark	Res	BL	BLRes	BLV	BLVR
Panel A. Low risk level						
Sharpe ratio	0.439	0.478	0.493	**0.534**	0.413	0.504
Farinelli-Tibiletti	0.336	0.342	0.339	**0.355**	0.338	0.353
Herfindahl index	0.435	0.392	0.373	0.337	0.333	**0.317**
Turnover	0.485	0.432	0.456	0.369	0.439	**0.347**
Panel B. Medium risk level						
Sharpe ratio	0.271	0.336	0.356	**0.442**	0.269	0.425
Farinelli-Tibiletti	1.339	1.403	1.370	**1.492**	1.361	1.491
Herfindahl index	0.297	0.204	0.228	**0.102**	0.201	0.114
Turnover	0.853	0.608	0.376	**0.299**	0.727	0.375
Panel C. High risk level						
Sharpe ratio	0.270	0.332	0.110	0.277	0.216	**0.342**
Farinelli-Tibiletti	6.718	7.139	5.357	5.970	6.462	**7.358**
Herfindahl index	0.347	0.183	0.372	**0.116**	0.328	0.126
Turnover	0.999	0.679	0.458	**0.359**	0.781	0.447

Notes: This table presents the results of the Sharpe ratio, Farinelli-Tibiletti ratio, Herfindahl index, and turnover for each methodology considering three levels of risk for the optimal portfolio: low (panel A), medium (panel B), and high (panel C). The estimation period considered was 60 months and the evaluation period was 12 months. Best results for each evaluation criterion are shown in bold.

Table 5
Best methodologies for sub-samples.

	Mark	Res	BL	BLRes	BLV	BLVR
FT	7	9	5	12	3	9
SR	7	10	2	15	2	9
HI	0	1	0	16	5	23
T	0	0	12	19	0	14
Total	14	20	19	62	10	55

This table shows how many sub-samples, considering three risk levels, each methodology was the best. We have in total 180 cases: 15 sub-samples, three risk levels and four performance evaluation criteria.

the Sharpe ratio, FT ratio and the Herfindahl index are better for the BLRes. The exception is turnover, which is better for the BL in the medium and high risk.

Finally, in the results for the BL with analysts' views (BLV) and the BL with analysts' views and resampling (BLVR) methodologies, the previous findings remain almost unchanged, with the BLVR outperforming in every dimension the BLV.

Thus, our BL-resampling combined methodologies (BLRes and BLVR) generates very competitive portfolios when compared to other methodologies, considering the three evaluation dimensions and the three levels of risk. Specifically for the low levels of risk, the best methodology is always one of our combined methodologies for all four performance measures.

We can highlight that, for the high-risk portfolios (panel C of Table 3), the low financial performance of the methodologies based on equilibrium (BL and BLRes). This might be due to the outlier position of the Japan equity portfolio in Fig. 1, since the equilibrium approach increases the expected returns of this asset because of its high risk. Thus, Japan equities enter into the optimization with a higher expected return, but realized returns would still be lower. In order to check whether this low performance is related to the presence of Japan equities in the sample, we ran the optimization exercise without Japan equities (not reported). Results for the high-risk portfolios are considerably different, and now favor BL and BLRes against Mark and Res. Thus there is evidence that these equilibrium methodologies are very sensitive to outlier's assets in terms of risk and return.

Table 9
Best methodologies grouped by geographic region and asset class type.

Asset type	Mark	Res	BL	BLRes	BLV	BLVR
All assets	0	6	4	8	1	17
North America	0	4	9	13	2	8
Europe/Asia–Pacific	4	5	0	15	1	11
Gov Bonds	0	0	3	19	6	8
Equities + US Govt	10	5	3	7	0	11
Total	14	20	19	62	10	55

Notes: This table shows how many times each methodology was the best, considering three risk levels, and detailed for each asset type and geographical region. We have in total 180 cases: 15 sub-samples, three risk levels and four performance evaluation criteria.

Table 6
Best methodologies grouped by time period and evaluation dimension.

	Full period						1986–1997						1998–2009					
	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR
FT	2	1	1	8	1	2	3	2	4	3	0	3	2	6	0	1	2	4
SR	2	2	0	10	0	1	4	1	2	5	0	3	1	7	0	0	2	5
HI	0	0	0	9	1	5	0	0	0	6	3	6	0	1	0	1	1	12
T	0	0	4	8	0	3	0	0	7	5	0	3	0	0	1	6	0	8
Total	4	3	5	35	2	11	7	3	13	19	3	15	3	14	1	8	5	29

Notes: This table shows how many times each methodology was the best, considering three risk levels, and detailed for each time period and evaluation criterion. We have in total 180 cases: 15 sub-samples, three risk levels and four performance evaluation criteria.

Table 7
Best methodologies grouped by time period and risk level.

	Full period						1986–1997						1998–2009					
	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR
Low	0	1	0	10	1	8	2	0	4	4	3	7	0	1	0	2	3	14
Medium	0	2	2	15	0	1	2	0	5	7	0	6	0	8	1	3	2	6
High	4	0	3	10	1	2	3	3	4	8	0	2	3	5	0	3	0	9
Total	4	3	5	35	2	11	7	3	13	19	3	15	3	14	1	8	5	29

Notes: This table shows how many times each methodology was the best, considering four evaluation criteria, detailed for each time period and risk level. We have in total 180 cases: 15 sub-samples, three risk levels and four performance evaluation criteria.

Table 8
Best methodologies grouped by risk level and evaluation dimension.

	Full period						1986–1997						1998–2009					
	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR
FT	1	0	2	6	1	5	3	2	4	3	0	3	2	6	0	1	2	4
SR	1	1	1	6	1	5	4	1	2	5	0	3	1	7	0	0	2	5
HI	0	1	0	0	5	9	0	0	0	6	3	6	0	1	0	1	1	12
T	0	0	1	4	0	10	0	0	7	5	0	3	0	0	1	6	0	8
Total	2	2	4	16	7	29	7	3	13	19	3	15	3	14	1	8	5	29

Notes: This table shows how many times each methodology was the best, detailed for each evaluation criterion and risk level. We have in total 180 cases: 15 sub-samples, three risk levels and four performance evaluation criteria.

Table 10
Value of the performance evaluation measures for low risk portfolios.

Assets	Performance criterion	Full sample						1986–1997						1998–2009					
		Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR
All assets	FT	0.336	0.342	0.339	0.355	0.338	0.352	0.548	0.576	0.562	0.591	0.569	0.610	0.187	0.192	0.179	0.198	0.200	0.207
	SR	0.539	0.573	0.562	0.606	0.526	0.589	1.080	1.134	1.114	1.207	0.160	1.215	−0.151	−0.105	−0.249	−0.104	−0.133	−0.056
	HI	0.446	0.402	0.377	0.353	0.337	0.336	0.380	0.343	0.335	0.287	0.232	0.259	0.464	0.420	0.481	0.428	0.386	0.369
	T	0.308	0.257	0.242	0.213	0.280	0.209	0.404	0.346	0.371	0.302	0.336	0.278	0.300	0.256	0.232	0.204	0.257	0.196
North America	FT	0.360	0.371	0.373	0.375	0.369	0.369	0.610	0.631	0.670	0.650	0.638	0.632	0.190	0.196	0.178	0.189	0.189	0.202
	SR	0.630	0.660	0.678	0.690	0.650	0.670	1.164	1.194	1.258	1.222	1.227	1.200	−0.139	−0.087	−0.152	−0.092	−0.118	−0.022
	HI	0.614	0.560	0.577	0.520	0.492	0.489	0.628	0.539	0.591	0.482	0.445	0.443	0.553	0.515	0.579	0.543	0.474	0.483
	T	0.206	0.184	0.212	0.175	0.191	0.157	0.293	0.269	0.391	0.279	0.245	0.220	0.237	0.210	0.177	0.177	0.245	0.160
Europe and Asia–Pacific	FT	0.314	0.320	0.318	0.334	0.312	0.317	0.457	0.458	0.414	0.466	0.448	0.446	0.259	0.267	0.246	0.272	0.242	0.272
	SR	0.357	0.380	0.373	0.415	0.372	0.387	0.726	0.786	0.665	0.830	0.775	0.772	0.125	0.162	0.070	0.162	0.080	0.172
	HI	0.274	0.263	0.243	0.252	0.251	0.240	0.270	0.260	0.223	0.246	0.220	0.226	0.265	0.253	0.285	0.268	0.274	0.261
	T	0.225	0.200	0.203	0.184	0.241	0.199	0.223	0.183	0.203	0.177	0.235	0.199	0.292	0.280	0.281	0.261	0.255	0.221
Gov Bonds	FT	0.367	0.369	0.366	0.372	0.367	0.366	0.423	0.437	0.464	0.458	0.442	0.450	0.225	0.227	0.209	0.219	0.252	0.248
	SR	0.553	0.561	0.559	0.589	0.582	0.574	0.627	0.671	0.764	0.780	0.712	0.733	0.031	0.044	0.017	0.026	0.141	0.120
	HI	0.642	0.613	0.599	0.600	0.563	0.576	0.601	0.579	0.597	0.587	0.513	0.554	0.660	0.626	0.613	0.635	0.591	0.586
	T	0.137	0.123	0.128	0.108	0.160	0.117	0.166	0.156	0.129	0.137	0.173	0.142	0.164	0.150	0.205	0.147	0.188	0.149
Equities + US Govt	FT	0.375	0.373	0.341	0.366	0.359	0.376	0.706	0.689	0.596	0.644	0.676	0.658	0.187	0.190	0.177	0.185	0.187	0.201
	SR	0.641	0.656	0.580	0.652	0.587	0.643	1.385	1.372	1.148	1.256	1.334	1.320	−0.179	−0.134	−0.229	−0.139	−0.164	−0.062
	HI	0.589	0.534	0.495	0.465	0.454	0.444	0.612	0.527	0.485	0.425	0.392	0.388	0.534	0.491	0.557	0.503	0.463	0.451
	T	0.230	0.198	0.225	0.178	0.239	0.171	0.291	0.260	0.361	0.260	0.300	0.209	0.262	0.214	0.188	0.169	0.255	0.168

Notes: This table shows the value of the performance evaluation measure for each sub-sample for the low risk level portfolios. We have in total 60 cases: 15 sub-samples and four performance evaluation criteria. For each case, we have the performance evaluation measure for all six methodologies.

It is also worth to mention that on Tables 3 and 4 the best methodology in terms of financial performance is always the same no matter the measure chosen is the Sharpe ratio or the FT ratio. Moreover, we used the three FT ratio's parameterizations for each risk level, and again the best methodology remained unchanged (not reported).

We show a first robustness test of our findings on Table 4, which presents the results with an evaluation period of 12 months, i.e., the optimizer considers the previous 5 years of data to estimate the frontier portfolios and holds the allocation for the next year, for which the returns are calculated. After that, the frontier is re-estimated again considering the previous 5 years of data and so on until the end of the sample period. Therefore, results of Table 4 are based on 23 portfolio optimization experiments for each methodology, since the holding period is 12 months.

The results are typically in line with the ones presented in Table 3, with resampling techniques typically improving results of the standard approaches. Our proposed methodologies (BLRes and BLVR) are still very competitive compared to the other methodologies, considering the three evaluation dimensions and the three levels of risk. In fact, for every evaluation dimension and risk level, the best methodology is either BLRes or BLVR.

In general, Sharpe ratios and Farinelli-Tibiletti ratios are smaller when compared with the results in Table 3. This is due to the fact that as we increase the evaluation period, the out-of-sample financial efficiency tends to decrease. In terms of diversification, the numbers are similar, so it seems that the methodologies are not affected in terms of diversification by the increase of the evaluation period. However, the turnover increased reasonably indicating that in order to reduce transaction costs, the portfolios should be re-estimated more often.

5. Robustness tests

In this section, we perform a series of robustness tests, in order to evaluate the consistency of previous results. We take our full sample and divide using as criteria the time period, geographical area and asset type. More specifically, we divide the time period into two: the first half from 1986 to 1997, and the second from 1998 to 2009. We also divide the sample considering two geographical areas: North America (US and Canada); Europe and Asia–Pacific. Finally we divide by asset type: one group with Government bonds only; and other with all Equities, US Corporate and the US Government.

We end up with 15 combinations for each of the three risk levels, summing up 45 optimization exercises for each methodology. For each of them, we calculated the four performance measures: Sharpe and FT ratios, HI and turnover. Tables 10 to 12 show all the performance numbers for every sample. In every optimization exercise we considered an estimation period of 60 months and an evaluation period of 6 months.

Table 5 presents the results on an aggregate level and shows the overall figures of all 45 sub-samples for each evaluation dimension. Each number in the table shows the number of times (sub-samples) where that methodology was the best[3] for that evaluation criterion. In the overall figures, including all evaluation dimensions, results are clearly favorable to our new combined methodologies BLRes and BLVRes. For the diversification (HI) dimension, the new methodologies are the best in 39 out of 45 cases. However, for the turnover (T) dimension, the BL is also competitive, along with the two new methodologies, which together were the best in 33 out of 45 cases. Resampling based methodologies, by explicitly considering the estimation error of expected returns and risk, typically provide more diversified and stable portfolios (Fernandes and Ornelas, 2009).

For the financial performance, BLRes has the best results in both Sharpe and FT ratios, but Resampling and BLVRes also present good performance. In roughly half of the cases, the best financial performance is either of BLRes or BLVRes. Regarding the resampling and Markowitz methodologies, we see that, although they are competitive in terms of financial performance, their results in terms of diversification and turnover are very poor. The results in favor of resampling based methodologies regarding financial performance are in line with other previous studies (see, for instance, Markowitz and Usmen, 2003; Fernandes and Ornelas, 2009).

[3] For each criterion, the "best" has different meanings: for the SR and FT, the best is the one with the highest number, for HI and T, the best is the one with the lowest number.

Table 11
Value of the performance evaluation measures for medium risk portfolios.

Assets	Performance criterion	Full sample						1986–1997						1998–2009					
		Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR
All assets	FT	1.339	1.403	1.370	1.492	1.361	1.491	2.082	2.225	1.864	2.266	2.257	2.408	0.940	1.004	0.620	0.915	0.904	0.983
	SR	0.367	0.428	0.402	0.509	0.388	0.503	0.969	1.066	0.799	1.090	1.094	1.186	−0.071	0.005	−0.570	−0.107	−0.116	−0.020
	HI	0.296	0.201	0.207	0.103	0.209	0.120	0.233	0.171	0.228	0.082	0.198	0.103	0.350	0.207	0.426	0.185	0.231	0.144
	T	0.577	0.380	0.191	0.196	0.494	0.256	0.610	0.412	0.211	0.234	0.528	0.286	0.747	0.434	0.481	0.298	0.587	0.319
North America	FT	1.518	1.553	1.573	1.647	1.527	1.595	2.521	2.467	2.832	2.693	2.428	2.452	0.948	1.047	0.755	1.022	0.767	0.959
	SR	0.547	0.575	0.598	0.652	0.560	0.607	1.306	1.292	1.446	1.413	1.280	1.305	−0.060	0.053	−0.315	0.025	−0.309	−0.047
	HI	0.430	0.338	0.362	0.215	0.358	0.230	0.474	0.377	0.339	0.195	0.399	0.224	0.384	0.286	0.470	0.293	0.328	0.242
	T	0.334	0.268	0.113	0.155	0.354	0.195	0.365	0.310	0.149	0.171	0.359	0.221	0.475	0.346	0.326	0.255	0.543	0.260
Europe and Asia–Pacific	FT	1.301	1.303	1.233	1.335	1.283	1.312	1.610	1.652	1.156	1.653	1.633	1.698	1.130	1.167	0.863	1.089	1.079	1.132
	SR	0.338	0.339	0.284	0.378	0.317	0.349	0.649	0.672	0.201	0.684	0.661	0.720	0.151	0.191	−0.184	0.111	0.094	0.157
	HI	0.317	0.263	0.226	0.176	0.253	0.176	0.303	0.240	0.219	0.170	0.214	0.157	0.283	0.233	0.317	0.216	0.280	0.194
	T	0.400	0.277	0.369	0.173	0.451	0.225	0.518	0.301	0.394	0.200	0.435	0.237	0.427	0.340	0.397	0.269	0.472	0.264
Gov Bonds	FT	1.416	1.437	1.492	1.554	1.458	1.503	1.302	1.359	1.614	1.644	1.389	1.508	1.135	1.135	1.073	1.128	1.267	1.245
	SR	0.453	0.474	0.533	0.594	0.490	0.545	0.358	0.422	0.639	0.684	0.435	0.567	0.159	0.158	0.093	0.156	0.305	0.288
	HI	0.448	0.380	0.376	0.319	0.312	0.291	0.366	0.307	0.341	0.287	0.253	0.250	0.467	0.392	0.493	0.380	0.373	0.322
	T	0.293	0.227	0.179	0.137	0.300	0.158	0.365	0.287	0.137	0.135	0.322	0.205	0.319	0.240	0.437	0.175	0.333	0.182
Equities + US Govt	FT	1.481	1.503	1.340	1.483	1.364	1.488	3.188	3.096	1.863	2.262	2.755	2.816	0.938	0.952	0.627	0.888	0.763	0.877
	SR	0.505	0.526	0.377	0.508	0.390	0.505	1.593	1.578	0.822	1.112	1.399	1.465	−0.077	−0.059	−0.556	−0.142	−0.309	−0.154
	HI	0.398	0.302	0.263	0.173	0.336	0.200	0.344	0.281	0.249	0.144	0.304	0.168	0.466	0.306	0.510	0.276	0.397	0.238
	T	0.445	0.322	0.201	0.181	0.482	0.226	0.460	0.346	0.201	0.202	0.494	0.249	0.608	0.424	0.289	0.369	0.678	0.292

Notes: This table shows the value of the performance evaluation measure for each sub-sample for the medium risk level portfolios. We have in total 60 cases: 15 sub-samples and four performance evaluation criteria. For each case, we have the performance evaluation measure for all six methodologies.

Table 12
Value of the performance evaluation measures for high risk portfolios.

Assets	Performance criterion	Full sample						1986–1997						1998–2009					
		Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR	Mark	Res	BL	BLRes	BLV	BLVR
All assets	FT	6.718	7.139	5.357	5.970	6.462	7.358	10.556	11.672	4.808	6.513	9.962	9.552	0.940	1.004	0.620	0.915	0.904	0.983
	SR	0.346	0.383	0.174	0.300	0.318	0.410	0.846	0.995	0.210	0.544	0.819	0.890	−0.071	0.005	−0.570	−0.107	−0.116	−0.020
	HI	0.344	0.180	0.339	0.113	0.330	0.127	0.318	0.164	0.493	0.134	0.356	0.127	0.350	0.207	0.426	0.185	0.231	0.144
	T	0.715	0.469	0.221	0.237	0.596	0.313	0.789	0.495	0.145	0.260	0.616	0.347	0.747	0.434	0.481	0.298	0.587	0.319
North America	FT	7.276	8.067	7.758	8.243	8.298	8.231	12.509	12.772	16.919	16.482	14.082	14.095	0.948	1.047	0.755	1.022	0.767	0.959
	SR	0.491	0.557	0.517	0.609	0.561	0.582	1.277	1.285	1.418	1.439	1.370	1.379	−0.060	0.053	−0.315	0.025	−0.309	−0.047
	HI	0.469	0.313	0.479	0.213	0.474	0.266	0.473	0.297	0.522	0.218	0.492	0.279	0.384	0.286	0.470	0.293	0.328	0.242
	T	0.460	0.311	0.115	0.189	0.320	0.236	0.429	0.332	0.170	0.191	0.274	0.225	0.475	0.346	0.326	0.255	0.543	0.260
Europe and Asia–Pacific	FT	6.790	6.388	4.880	5.869	5.410	6.294	7.759	7.793	3.745	6.134	5.351	7.132	1.130	1.167	0.863	1.089	1.079	1.132
	SR	0.358	0.311	0.163	0.273	0.184	0.268	0.612	0.581	−0.116	0.381	0.257	0.546	0.151	0.191	−0.184	0.111	0.094	0.157
	HI	0.448	0.271	0.370	0.146	0.338	0.148	0.438	0.226	0.443	0.146	0.330	0.138	0.283	0.233	0.317	0.216	0.280	0.194
	T	0.585	0.371	0.544	0.213	0.608	0.286	0.789	0.434	0.565	0.258	0.572	0.302	0.427	0.340	0.397	0.269	0.472	0.264
Gov Bonds	FT	6.300	6.318	6.788	6.559	6.266	6.576	5.579	5.791	6.609	6.860	5.803	6.307	1.135	1.135	1.073	1.128	1.267	1.245
	SR	0.322	0.345	0.444	0.527	0.353	0.495	0.123	0.213	0.460	0.545	0.151	0.352	0.159	0.158	0.093	0.156	0.305	0.288
	HI	0.485	0.302	0.412	0.210	0.361	0.211	0.440	0.292	0.321	0.197	0.371	0.215	0.467	0.392	0.493	0.380	0.373	0.322
	T	0.477	0.346	0.301	0.185	0.402	0.227	0.580	0.399	0.226	0.173	0.397	0.262	0.319	0.240	0.437	0.175	0.333	0.182
Equities + US Govt	FT	7.757	7.461	5.072	6.131	6.961	7.199	18.789	15.232	4.862	7.148	13.503	11.656	0.938	0.952	0.627	0.888	0.763	0.877
	SR	0.483	0.473	0.109	0.319	0.391	0.411	1.577	1.537	0.218	0.610	1.278	1.170	−0.077	−0.059	−0.556	−0.142	−0.309	−0.154
	HI	0.375	0.233	0.360	0.148	0.381	0.172	0.325	0.195	0.484	0.162	0.374	0.173	0.466	0.306	0.510	0.276	0.397	0.238
	T	0.606	0.405	0.241	0.213	0.498	0.279	0.630	0.403	0.151	0.243	0.476	0.301	0.608	0.424	0.289	0.369	0.678	0.292

Notes: This table shows the value of the performance evaluation measure for each sub-sample for the high risk level portfolios. We have in total 60 cases: 15 sub-samples and four performance evaluation criteria. For each case, we have the performance evaluation measure for all six methodologies.

On the other hand, the equilibrium BL has a good performance in terms of turnover, but fails to be competitive in terms of financial performance. The BLV seems to be the worst method in our empirical evaluation. Then we conclude that the combined methodology proposed was able to incorporate both the good financial performance of Mark and Res and the good low turnover of BL.

Below we deepen our analysis by segmenting results by time period, and grouping by evaluation criteria, geographical area, asset class, and risk level. The numbers in the following four tables are again the number of times (sub-samples) where that methodology was the best.

Table 6 shows results segmented by time period and evaluation dimension. For each time period, each row has 15 evaluations: 3 risk levels times 5 sub-samples (the full sample, two geographical areas and two asset types groups). Overall figures for this table show an advantage to BLRes, especially in the full sample period, which is the most representative. In the full sample period the BLRes is the best methodology in all four evaluation criteria. For the diversification (HI) dimension, the two new combined methodologies (BLRes and BLVRes) have clearly the best results in all periods. In the turnover dimension, BLRes is better in the Full period, while BLVRes tops in the second half. However, during the first half BL was the best. For the financial performance dimension, BLRes is the best in the full sample period. For the first period sample, there is not a clear winning model in terms of financial performance. For the second half of the time period, Resampling has a better result in terms of financial performance, although BLVRes is still competitive.

Table 7 shows results grouped by risk level and segmented by time period. For each time period, each row has 20 evaluations: 4 evaluation criteria times 5 sub-samples (the full sample, two geographical areas and two asset types groups). In this case, the meaning for the "best" is the methodology with the highest number of best values considering all criteria (FT, SR, HI and T). So we actually sum how many times the methodology is the best across different criteria. The BLRes was the best in the full period, for all risk levels. When we analyze the time period sub-samples, we see that BL and BLRes are competitive in the first half, and in the second half BLVRes is by far the best.

Table 8 shows results by risk level and evaluation dimension. For each risk level, each row has 15 evaluations: 3 evaluation periods times 5 sub-samples (the full sample, two geographical areas and two asset types groups). For the diversification and turnover dimensions, BLRes and BLVRes have clearly a better result in all risk levels. For the financial performance dimension, results vary according to risk levels. For the low risk level, the two new combined methodologies (BLRes and BLVRes) are the best. For the medium level of risk, the dispute for the first place is between Res and BLRes. For the high risk level, Mark and Resampling have the best results in terms of financial performance.

Finally, Table 9 presents the result by asset class and geographical area. Each row has 36 evaluations, which is the combination of three risk levels, three time periods and four evaluation criteria. Table 9 is similar to Table 5, with the difference that in Table 5 results are broken down by evaluation criteria, while in Table 9 they are broken down by geographic region and asset class. Thus, the last lines of Tables 5 and 9 have the same totals. Results show that for both geographic regions BLRes has a better performance. BLRes is also the best for the Government Bonds sample. For the Equities plus US Government bond sample, BLVRes and Markowitz offer the best results. The BLVRes is also the best when we use all assets.

The general conclusion of the robustness tests of this section is that our proposed methodologies roughly outperform the remaining ones, even considering several samples with different grouping criteria.

6. Conclusion

This paper deals with a well-documented issue in mean–variance optimization, related to the fact that this methodology typically leads to unintuitive portfolios with extreme positions in asset classes. In this article, we proposed the use of an optimization approach that takes advantage of both BL and resampling techniques to incorporate the main positive aspects of both previous powerful techniques. It is a stochastic general equilibrium model, which can be used as a tool for both passive and active strategies. The main idea is to estimate the efficient frontier using the BL model but consider this frontier as just an input to the resampling method.

We empirically test this methodology using a comprehensive sample of bond and stock indices. Compared to traditional portfolio optimization methodologies, we reached very supportive results. We found strong evidence supporting the use of resampling techniques to improve standard methodologies. Generally speaking, our proposed methodologies, both with and without views, generated very competitive portfolios compared to the other methodologies, considering the three evaluation dimensions: financial efficiency, diversification, and allocation stability; and several levels of risk.

As a suggestion for further research, we would recommend the use of non-normal distributions, instead of the multivariate normal, together with the BL and Resampling methodologies. For instance, we may extend the MAGH (Multivariate Affine Generalized Hyperbolic) portfolio optimization approach of Fajardo and Farias (2009) in a way to include equilibrium returns and resampling.

It is important to highlight that recommendation of specific analysts' views methodologies is out of the scope of the present study. The view considered in this article is just a naive example to show that the proposed methodology may be adapted to the analysts' views. We argue that the proposal of views methodologies is still an open avenue for future research in portfolio management.

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